Re-Exam

Autumn 2018

Important: Please make sure that you answer all questions and that you properly explain your answers. For each step write the general formula (where relevant) and explain what you do. Not only the numerical answer. If you make a calculation mistake in one of the earlier sub-questions, you can only get points for the following subquestions if the formula and the explanations are correct!

1. Short questions.

   (a) Order the solution concepts based on their strength from weakest to strongest: IESDS, Nash Equilibrium, Perfect Baysian Equilibrium, Subgame Perfect Nash Equilibrium

   Solution: IESDS, Nash Equilibrium, Subgame Perfect Nash Equilibrium, Perfect Bayesian Equilibrium

   (b) "Cheap-talk is never useful because it doesn't affect the players outcomes" True or False? What are the three requirements for cheap talk to be informative? Explain in 2-3 sentences.

   Solution: False! Cheap talk can be useful when trying to coordinate on an outcome that benefits both individuals or at least doesn't make the sender worse off. 1) Different sender-types prefer different actions. 2) The receiver prefers different actions for different sender-types. 3) Sender types' and receivers' preferences are not completely opposed.

   (c) Consider a third-price auction of a single object (the highest bidder gets the object and pays the third highest bid; the others do not pay). Give an example (values and bids) that shows that truthful bidding is not a dominant strategy in this auction, and explain your example in 2-3 sentences.

   Solution: Suppose there are three bidders with bids b1 = 10, b2 = 5, and b3 = 0, and bidder 2 has a value of v2 = 5. Given these bids, bidder 2 has no incentive to bid truthfully at b2 = 5 since by bidding b2 > 10 he wins the object, pays 0 and gets a net payoff of 5, which is larger than his payoff of 0 from truthful bidding.

   (d) It can happen that there is no core in a coalition game. True or False?

   Solution: False! The core is a set of actions of the grant coalition N. Because the core is defined as a set of actions it always exists, but it can be an empty set.

   (e) In many games the order of play matters. Give an example of a game with a first-mover advantage and a game with a last mover advantage and explain your reasoning in 2-3 sentences.

   Solution: Last mover advantage are games like zero-sum games and all dynamic games that allow the second player to profitably react to the first players choice. First mover advantage is possible in many bargaining games or Cournot-Stackelberg.

2. Consider the following game

		Player 2		
		X	Y	Z
	A	$4, 4$	$0, 2$	$4, 5$
Player 1	B	$0, -1$	$3, 3$	$3, 2$
	C	$3, 3$	$-1, 1$	$1, 4$

(a) Find all Nash Equilibria (pure and mixed) of G. Let p = probability that the row player plays A. q = probability that the column player plays Y.

> **Solution:**
> Pure strategy: (B,Y) and (A,Z) In the mixed equilibrium players need to be indifferent between the pure strategies with which they mix. Strictly dominated strategies are never played with positive probability. Since C and X are strictly dominated, they will never be played in equilibrium. So the players will mix between the other strategies. A,B and Y,Z.
>
> $$0q + 4 - 4q = 3q + 3 - 3q$$
>
> $q = 1/4$
>
> $$2p + 3 - 3p = 5p + 2 - 2p$$
>
> $p = 1/4$
> So the mixed strategy NE is ((1/4),(3/4),0), (0,(1/4)(3/4))

(b) In which of the equilibria you found in a) does player 1 have the highest expected payoff?

> **Solution:** Player 1 has a payoff of 4 from (A,Z) and a payoff of 3 from (B,Y). From the mixed-strategy NE, 1's expected payoff is $39/16 = 2.43$. He therefore gets the highest expected payoff from the pure-strategy NE (A,Z). [The answer is not complete if the expected payoff from the mixed-strategy NE is not calculated.]

(c) Use iterated elimination of strictly dominated strategies on G and check that all pure strategy NE you found in a) survived IESDS. Is it possible that a NE does not survive IESDS? Explain briefly.

> **Solution:** X is strictly dominated by Z and C is strictly dominated by A. Both pure NE survive IESDS. It is not possible that a NE does not survive IESDS. That would mean that one of the strategies played in the NE is strictly dominated, but by definition all strategies played in a NE are best responses to each other.

3. Consider the infinitely repeated game with a discount factor δ, where the stage game (G) looks like this:

		Player 2	
		L	R
Player 1	U	$1, 1$	$-2, 5$
	D	$2, 0$	$0, 0$

(a) What is the Nash equilibrium of the stage game (G)? What are the players' payoffs in the NE? Could the players do better by coordinating on a Nash threat (trigger) strategy? Under what condition would they do better?

> **Solution:** The NE of the stage game is (D,R) giving payoffs of (0,0). (D,R) is the correct Nash equilibrium here, because R weakly dominates L and D strictly dominates U. Yes, they could do better by playing a Trigger strategy.

(b) Write down the simplest possible trigger strategy, for each player, such that the following holds: if both players stick to their trigger strategies, then the outcome will be (U,L) in every period. Show that for sufficiently patient players, the strategy profile where both players play trigger strategies constitutes an SPNE. What is the minimum value of delta for which this strategy profile is an equilibrium?

> **Solution:** If the history is (U,L), (U,L)...(U,L) play (U,L) this period. After any other history, play (D,R).
>
> Does either player have a profitable deviation? For the row player following the proposed strategies gives a payoff of
>
> $$1 + \delta 1 + \delta^2 1 + ... = \frac{1}{1-\delta}$$
>
> and deviating gives a payoff of
>
> $$2 + \delta 0 + \delta^2 0 + ... = 2$$
>
> so cooperating is better than deviating if
>
> $$\frac{1}{1-\delta} \geq 2$$
>
> $$\delta \geq 1/2$$
>
> For the column player, following proposed strategies gives a payoff of
>
> $$1 + \delta 1 + \delta^2 1 + ... = \frac{1}{1-\delta}$$
>
> and deviating gives a payoff of
>
> $$5 + \delta 0 + \delta^2 0 + ... = 5$$
>
> so cooperating is better than deviating if
>
> $$\frac{1}{1-\delta} \geq 5$$
>
> $$\delta \geq 4/5$$
>
> Therefore, as long as
>
> $$\delta \geq 4/5$$
>
> the trigger strategies are a Subgame Perfect Nash Equilibrium of the infinitely repeated game.

4. Two firms compete in the market for gløgg, where Firm 1 produces red wine based gløgg and Firm 2 produces white wine based gløgg. These products are imperfect substitutes: demand for red wine gløgg is: $q_1 = 1 - p_1 + p_2$, and demand for white wine is $q_2 = 1 - p_2 + p_1$, where p_1 and p_2 are the prices of Firm 1 and 2. Firm 1 has access to new wine barrels that allows it to produce at zero marginal cost. Firm 2 is still using old-fashioned barrels, so it has marginal costs of $c > 0$. This means profits for Firm 1 are $\pi_1 = q_1 p_1$, and profits for Firm 2 are $\pi_2 = q_2(p_2 - c)$. Firms set prices simultaneously and independently.

   (a) Show that in the Nash Equilibrium of this game, firms set prices $p_1 = 1 + c/3$, $p_2 = 1 + 2c/3$. Explain intuitively why both equilibrium prices are increasing in c. (2-3 sentences).

   Solution: Taking the first-order condition gives best-response functions: $p_1 = (1 + p_2)/2$ and $p_2 = (1 + p_1 + c)/2$, which yields equilibrium prices $p_1 = 1 + c/3$, $P_2 = 1 + 2c/3$. Firm 2s optimal price is directly increasing in its cost, since high costs mean lower revenues from every sale. Firm 1's optimal price is indirectly increasing in Firm 2's cost because of strategic complementarities. When firm 2 sets a high price, this pushes Firm 1 to set a high price as well, since best response functions are upwards sloping.

   (b) Suppose Firm 2 develops an innovation that it hopes will lower production costs. If the innovation works, then Firm 2's marginal costs become zero. If the innovation does not work, then its marginal costs remain at c. Firm 2 knows whether or not the innovation works (so it knows its own marginal cost), but Firm 1 does not. Firm 1 believes there is a probability 1/2 that the innovation works, so that Firm 2 has marginal cost of zero, and a probability 1/2 that it does not work, so that Firm 2 has marginal cost of c. Write down the three best-response functions that, taken together, implicitly define the prices in the Bayes-Nash equilibrium of this game. (Bonus points: Solve for the equilibrium prices.)

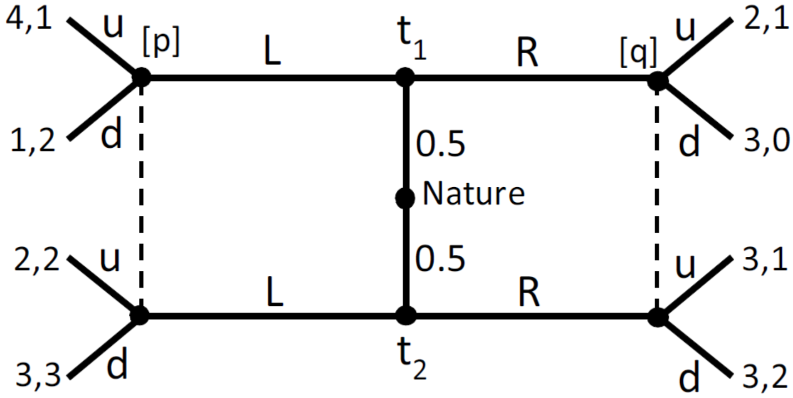
   Solution: From part a) we know that the best response function for Firm 2 is $p_2^H = (1 + p_1 + c)/2$ if it is a high cost type. Setting c=0 the best response function is $p_2^L = (1 + p_1)/2$ if it is a low cost type. Expected demand for firm 1 is $(1/2)1 - p_1 + p_2^H + (1/2)1 - p_1 + p_2^L = 1 - p_1 + (p_2^H + p_2^L)/2$.
   Its best response function is $p_1 = (1 + (p_2^H + P_2^L)/2/2)$. Bonus points: Solving these three linear equations in three unknowns gives the equilibrium prices: $p_1 = 1 + c/6, P_2^L = 1 + c/12, p_2^H = 1 + 7c/12$.

   (c) Imagine that before setting prices, Firm 2 announces to Firm 1: "Unfortunately, the innovation does not work, so I have high costs!" Why might Firm 2 make such an announcement? How do you expect this announcement to affect the price set by Firm 1? What might Firm 2 do instead, to convince Firm 1 that it has high costs? Explain your answers briefly (2-3 sentences each).

   Solution: Firm 2 would like to convince Firm 1 that it has high costs, since part (a) shows that Firm 1 would then set a higher price. This would increase demand for Firm 2, allowing it to earn higher profits. Firm 1 should understand that Firm 2's announcement is not credible, since it would have an incentive to make this announcement (if it were believed) regardless of its actual costs. In equilibrium, Firm 1 ignores the announcement, Firm 2 realizes it will be ignored, and they set the same prices as in part (b).

Firm 2 could burn down their new barrels to show that the innovation was not successful.

5. Have a look at the following signaling game G"



(a) Is this a game of complete or incomplete information?

Solution: By definition G" is a game of incomplete information.

(b) Find all separating Perfect Bayesian Equilibria.

Solution: The only separating PBE is: [(R; L); (d; u); p = 0; q = 1] I.e., type 1 sends the message R, type 2 sends the message L. After seeing the message L, the receiver chooses the action d. After seeing the message R, he chooses the action u.